LESSON PLAN · §U3.1

Work and the work-energy theorem

Sixty minutes on one question: which piece of the force lies along the motion, and which way does it point.

AP Physics 1 · 60 min, break at 30 · Giancoli, Physics: Principles with Applications, 6th ed., sections 6-1 through 6-3. CED Unit 3: Work, Energy, and Power. Prerequisites: free-body diagrams, components on an incline, and v-squared equals v-zero-squared plus 2ad.

STUDENT PROFILE NOTE

Reusable master. Adapt names and numbers, keep the sequence. This lesson is built against four specific errors, in the order they cost the most marks. One, writing work as force times distance and never asking about the angle, which credits the normal force with doing work and puts the full weight into an incline calculation instead of the component along the motion. Two, treating work as a vector and drawing it on the free-body diagram, which is the same error as labelling an arrow centripetal force. Three, giving friction positive work, which produces a larger final speed and no signal that anything went wrong. Four, learning the theorem as a fifth formula rather than as the second law rewritten over a distance, after which the student never reaches for it on the one problem type it exists for, force unknown and distance known. If the hour runs long, the cut list is marked in each phase. Do not cut the derivation in the Application phase and do not cut problem 6.

COMMIT BEFORE YOU COMPUTE | 0 TO 8 MIN

Warmup

Open by naming the shape of the hour out loud. Something like: eight minutes on four quick questions you already have opinions about, twenty minutes building one equation, a break, then the theorem and problems. Saying it costs fifteen seconds and it stops the session feeling improvised.

Then run problem 1 as a prediction round. The student writes an answer to all four parts before you say anything about any of them. This matters: the memory effect that makes this warmup work needs a committed prediction, not a shrug. Do not let them answer out loud one at a time and drift toward whatever you seem to want.

1.

For each situation, is the work done by the named force positive, negative, or zero? Write all four answers down before we discuss any of them.

(a) You carry a heavy bag at constant speed 10 m across a level floor. Consider the upward force of your hand on the bag.
(b) You stand still holding the same bag for thirty seconds. Same force.
(c) You are standing in an elevator that is rising from the ground floor to the third floor. Consider the normal force of the floor on you.
(d) The Moon orbits the Earth in a nearly circular orbit. Consider the gravitational force of the Earth on the Moon.

Giancoli, ch. 6, Fig. 6-2 and Conceptual Example 6-3, plus end-of-chapter Questions 2 and 3. Assembled here as a prediction round.

SOLUTION

(a) Zero. The hand pushes up, the bag moves sideways. The angle is 90 degrees and cos 90 is 0.

(b) Zero. There is a force but no displacement. Work needs both.

(c) Positive. The normal force points up and you moved up. The angle is zero. This is the one that surprises people. Send the elevator down instead and the same normal force does negative work, which is worth saying out loud, because what set the sign was the direction you travelled and nothing else.

(d) Zero. Gravity points at the centre of the orbit, the Moon's displacement at every instant is tangent to the circle, and those are perpendicular. This is why satellites stay in orbit without burning fuel.

COMMON WRONG PATH

Part a answered positive because carrying a heavy bag across a room is tiring. That is a real observation about muscles and it is not work in the physics sense. Do not dismiss it. Say that their arm is doing work internally on itself, and that the quantity we are defining tracks something else, which is whether the bag speeds up. Part c answered zero by reflex, from a half-remembered rule that normal forces do no work.

IF THEY STALL, ASK

"Point at the force. Point at where the object went. Are those two arrows in the same direction, opposite directions, or at right angles?"

> TUTOR NOTE
> Collect all four before revealing any. Then reveal in this order: b, a, d, c. Parts b and a are the easy wins and they establish that the answer is allowed to be zero. Part d gets a nod from anyone who has done circular motion. Part c is the one that breaks the rule they just built, which is exactly why it is last. The sentence to get out of them after c, in their own words: the normal force is not special. Nothing about being a normal force makes work zero. Every case they had seen was an object moving horizontally, where the normal force happens to sit at 90 degrees to the motion. Here the motion is vertical and the same force lies straight along it. The force did not change. The direction of travel did. If they say 'the normal force never does work' at any point in the hour, this is the counterexample to hand back.

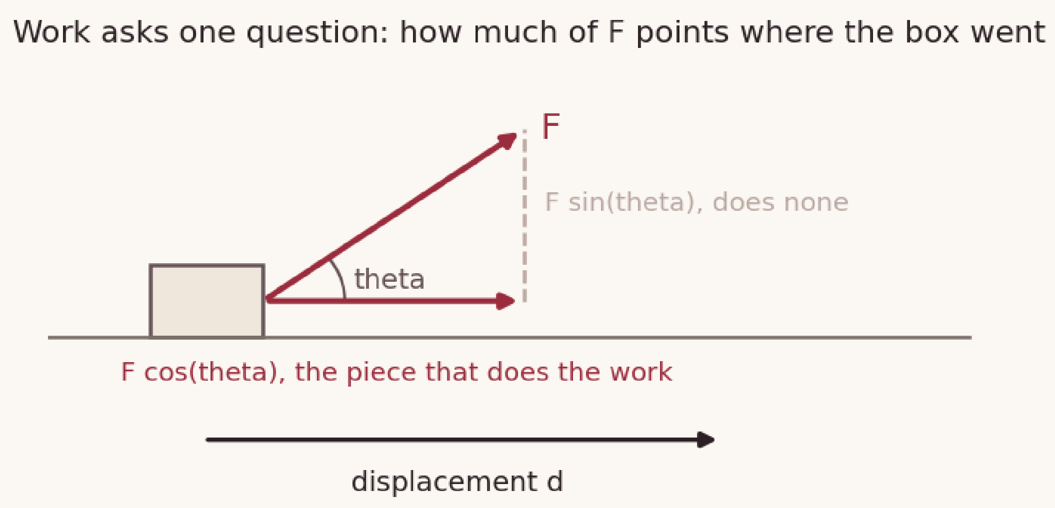
Close the warmup with the sentence the rest of the hour hangs on, and write it where it will stay visible: a force does work on an object only to the extent that it points along the direction the object actually moved. Four situations, one rule, and they produced it themselves. Do not define work yet. The equation comes next and it should read as a way of writing down what they just said.

BUILD THE DEFINITION, THEN THE SIGN | 8 TO 30 MIN

Conceptual

Roughly seven minutes on the projection and the equation, six on the sign, seven on the incline, and two on what work is not. The student holds the pen for the incline decomposition. If you are talking for more than about four minutes at a stretch in this phase, stop and hand over the pen.

Draw a box on a floor with a rope pulling at an angle, and ask them to split that pull into two arrows, one along the floor and one straight up. They have done this decomposition since the first week of the course. The only new claim is which piece counts.



The upward piece is not small. It is irrelevant. The box never moved upward, so that piece has nothing to multiply against.

Now write the definition, and write it in the order that matches the picture rather than in the order it appears on the formula sheet.

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W  =  (F cos theta) d      the piece of F along the motion
W  =  F d cos theta        same thing, rearranged
1 joule  =  1 newton x 1 metre
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Then say what theta is, slowly, because this is where the whole error lives: theta is the angle between the force and the displacement. Not the angle in the picture, not the angle of the ramp, not whatever angle the problem happens to mention. Ask them to say it back.

2.

A person pulls a 50 kg crate 40 m along a horizontal floor with a constant 100 N force at 37 degrees above the horizontal. The floor is rough and exerts a 50 N friction force. Find the work done on the crate by each of the four forces acting on it.

Giancoli, Example 6-1, ch. 6.

SOLUTION

Four forces, so four calculations. Two of them are one line each.

Gravity: perpendicular to the displacement, so W = 0.
Normal force: perpendicular, so W = 0.
The pull: W = (100)(40) cos 37 = 3200 J.
Friction: points opposite the motion, so theta is 180 degrees, and W = (50)(40)(cos 180) = (50)(40)(-1) = -2000 J.

Net work: 0 + 0 + 3200 - 2000 = 1200 J.

COMMON WRONG PATH

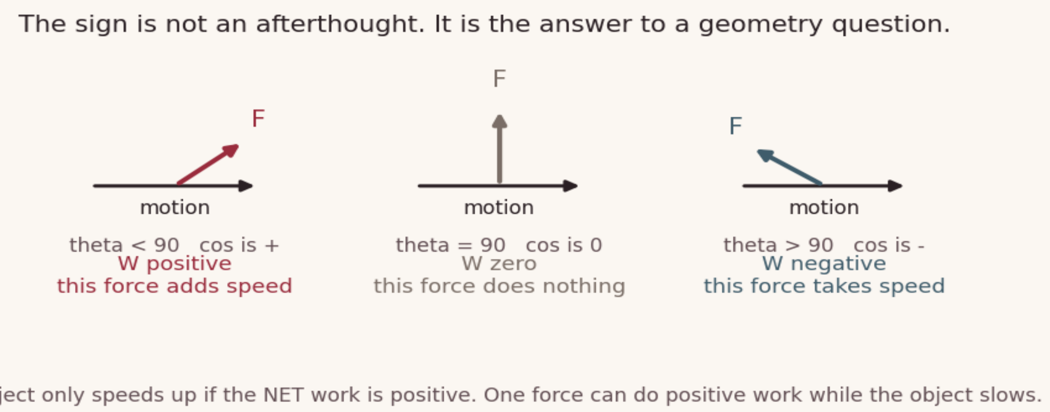
W for the pull computed as (100)(40) = 4000 J, dropping the cosine. Ask how far the crate moved in the direction the rope was pointing. Second common path: computing the work done by the normal force as (490)(40) because both numbers are available and multiplying them is easy.

IF THEY STALL, ASK

“What angle does each of these four arrows make with the arrow showing where the crate went?”

> TUTOR NOTE
> Have them list the four forces before computing anything. Most students find three and forget the normal force, precisely because it does no work, which is a good moment to point out that they cannot know it does no work until they have found it and looked at its angle. Ask for the friction angle explicitly. Do not accept 'friction is negative because friction is always negative.' The reason is that theta is 180 degrees and cos 180 is minus one. That reason will still be right in three weeks when they meet friction that does positive work, and the rule will not be. If they are moving fast, add: the crate started from rest, so how fast is it going after 40 m? They cannot answer yet. Leave it hanging. It gets answered in the Application phase and the answer is 6.9 m/s.

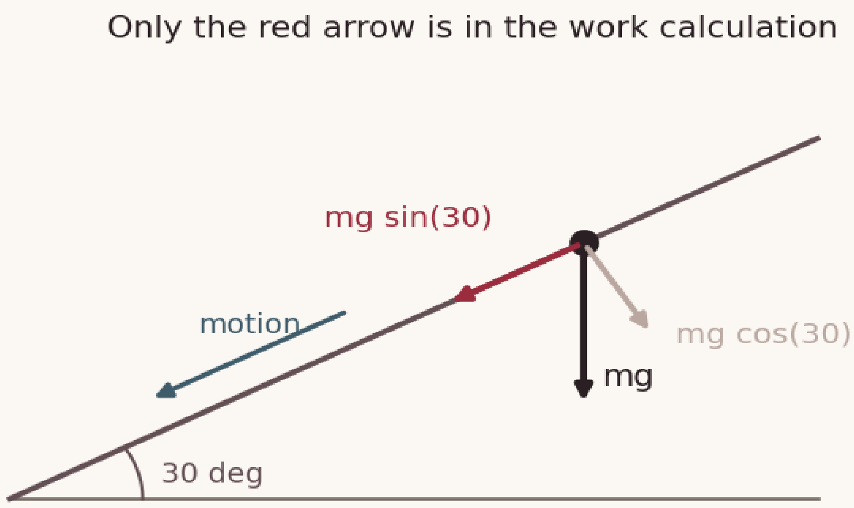
Now the sign, and this is where the hour should slow down. Ask before you show: cosine is positive for angles under 90 degrees and negative above. What does a negative work mean physically? Get them to answer it in terms of speed rather than in terms of the equation. Then hold the qualifier in place. A force doing negative work is taking speed away. Whether the object actually slows down depends on what the other forces are doing at the same time.



The angle between a force and the displacement sets the sign of the work that force does. Positive net work speeds the object up, negative net work slows it down, zero net work leaves the speed alone. Say net every time. A 10 N push against 20 N of friction does positive work on an object that is slowing down.

Then the point that has to be said out loud, because it is the one every student gets wrong in writing later: work is a scalar. It has a sign, but a sign is not a direction. Negative work does not mean work pointing backwards. There is no such thing as the direction of the work done by friction. It cannot be drawn as an arrow, and it does not go on a free-body diagram, because a free-body diagram shows forces and work is not a force. Temperature is negative in January and nobody draws an arrow for it.

The incline. Hand over the pen for this one. A block on a 30 degree ramp, sliding down. Ask them to decompose the weight and then tell you which piece appears in the work calculation.



The block moves along the ramp, so only the component along the ramp has anything to multiply against. The perpendicular component is what the normal force cancels.

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W by gravity  =  (mg sin 30) d        component along the motion
              =  mg d cos 60        angle between mg and d is 60 deg
                                       same number, two routes
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Both routes are worth writing, because students who learn only the second one get confused by the 60 when the ramp says 30, and students who learn only the first cannot handle a force that is neither along nor perpendicular to the motion. Then ask the question that ends the phase: how far did the block fall, vertically? Answer, d sin 30. So the work done by gravity is mg times the height dropped, and the angle of the ramp does not appear in that sentence at all. That result is the seed of gravitational potential energy in the next lesson.

THE THEOREM, DERIVED. BREAK AT 30, RESUME HERE | 32 TO 48 MIN

Application

Take the break at the 30 minute mark, before this phase and not inside it. The derivation below is four lines and it does not survive being cut in half.

Open after the break with the question, not the result: you already have three ways to find a final speed. Why would anyone want a fourth? Leave that hanging for the two minutes the derivation takes.

3.

Take a constant net force acting on a mass m over a straight-line displacement d, starting at speed v-zero and ending at speed v. You already know Newton's second law and you already know v-squared equals v-zero-squared plus 2ad. Combine them to get an expression for the net work that contains no acceleration and no time.

Giancoli, ch. 6, section 6-3, derivation of eq. 6-2. Run as a live derivation.

SOLUTION

Start from the definition of net work with the force along the motion:

W-net = F-net d = (ma) d

Solve the kinematic relation for a:

a = (v squared minus v-zero squared) / (2d)

Substitute:

W-net = m d (v squared minus v-zero squared) / (2d)

The d cancels, and what is left is

W-net = one half m v squared minus one half m v-zero squared

COMMON WRONG PATH

Substituting into the wrong kinematic equation and ending up with a t that will not leave. If they reach for d = v-zero t plus one half a t squared, let them go two lines, then ask which of the kinematic equations has no time in it to begin with.

IF THEY STALL, ASK

“You want a gone and t gone. Which of the four kinematic equations already has no t in it?”

TUTOR NOTE

They do this, you do not. Give them the two starting equations and stay quiet. It takes most students two to three minutes and the cancellation of d is the moment worth waiting for. When it lands, say what actually happened: nothing new was assumed. This is F equals ma multiplied through by a distance. It is the second law rewritten so that distance is the variable instead of time. That framing is the entire reason this phase exists, because a student who thinks the theorem is a fifth formula will never reach for it, and a student who knows it is the second law over a distance will reach for it exactly when a distance is what the problem gave them. Then name the quantity. One half m v squared is the kinetic energy. Notice that it fell out of the algebra rather than being handed down, and that it is quadratic in the speed. Double the speed and the kinetic energy goes up by four, which is why braking distance goes up by four.

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W-net  =  KE-final  -  KE-initial          KE  =  (1/2) m v^2

net, meaning the work done by EVERY force. Not the applied force.
              Not the biggest force. Every force.
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Underline the word net twice. The single most common misuse of this equation all year is setting the work done by one force equal to the change in kinetic energy. Every problem where an object is lifted, or dragged against friction, has at least two forces doing work, and the theorem is about their sum.

4.

Back to the crate from problem 2. It started from rest and 1200 J of net work was done on it over the 40 m. How fast is it moving at the end?

Continuation of Giancoli Example 6-1.

SOLUTION

W-net = one half m v squared minus zero, so 1200 = one half (50) v squared.

v squared = 48, and v = 6.9 m/s.

COMMON WRONG PATH

Using 3200 J, the work done by the pull, instead of the net 1200 J.

IF THEY STALL, ASK

“Which work goes into the theorem?”

TUTOR NOTE
Two sentences, and its whole job is to close the loop you left open before the break. If you did not leave it open, skip this and go to problem 5. It also gives the first chance to say that the answer would have been wrong if friction had been entered as positive, and by how much: 5200 J of net work gives 14.4 m/s, more than twice as fast, from one sign.

5.

An 88 g arrow is fired from a bow whose string exerts an average force of 110 N on the arrow over a distance of 78 cm. What is the speed of the arrow as it leaves the bow?

Giancoli, ch. 6, problem 19.

SOLUTION

W = (110)(0.78) = 85.8 J, and the force is along the motion so the cosine is one.

The arrow starts at rest, so 85.8 = one half (0.088) v squared, giving v squared = 1950 and v = 44 m/s.

COMMON WRONG PATH

Leaving 78 cm in centimetres and getting 8580 J, then a speed of 440 m/s. Nothing about the number looks obviously wrong to a student who has never seen an arrow. Ask whether an arrow travels faster than a rifle bullet.

IF THEY STALL, ASK

“What does the problem give you, and which relation connects those quantities to a speed?”

TUTOR NOTE
Two lines, and the second one is the trigger pattern. Do not claim this problem is impossible by any other route. It is not. The acceleration is 110 divided by 0.088, which is 1250 m/s squared, and v squared equals 2ad gives the same 44 m/s. If a student finds that, say so, and use it: the theorem got there without ever computing 1250, which is a number nobody wanted. For the case where the second law genuinely runs out, use Giancoli Example 6-5. A 1000 kg car goes from 20 m/s to 30 m/s and the question is the net work. No distance, no time, so there is no acceleration to be had and F equals ma has nothing to stand on. The theorem answers it in one line. The trigger pattern, worth writing on the page as a sentence they can look up later: speeds and a distance in the problem, time absent and unwanted. Reach for the theorem.

6.

A 2.5 kg block starts from rest and is pushed 4.0 m along a level floor by a constant horizontal 12 N force. Friction on the block is a constant 4.0 N. A student computes the net work as (12)(4.0) plus (4.0)(4.0) equals 64 J and gets a final speed of 7.2 m/s. Find the error, give the correct speed, and then find a way to catch this mistake that does not involve redoing the calculation.

Original problem, written for this lesson. This is problem 7 on the homework set.

SOLUTION

Friction opposes the motion, so its work is negative: -16 J, not +16 J. The net work is 48 - 16 = 32 J, and one half (2.5) v squared = 32 gives v = 5.1 m/s.

The check. Take friction away completely. Then the net work is 48 J and the speed is 6.2 m/s. The student's answer of 7.2 m/s is faster than the block would go on a frictionless floor. A rough floor cannot beat a smooth one. The answer was wrong before any algebra was rechecked.

COMMON WRONG PATH

Finding the sign error and stopping. Half the value of the problem is in the second half of the question. Make them produce the frictionless bound.

IF THEY STALL, ASK

“If the floor were perfectly smooth, would the block end up faster or slower than 5.1 m/s? Compute that number.”

> TUTOR NOTE
> Do not cut this problem. It is the one that changes behaviour rather than adding knowledge. Give them the check to find, do not hand it over. Ask: is there any number in this problem you can compute in your head that the answer has to be smaller than? Most students need one nudge, which is 'what if the floor were ice?' The reason to spend real minutes here is that a sign error in work has no local symptom. The units are joules, every number is positive, the arithmetic is right, the answer looks like an answer. Compare with a sign error in kinematics, which usually produces a negative square root or a car that arrives before it left. This one is silent, so the student has to supply the alarm themselves.

ONE COLD SOLVE, THEN CLOSE | 48 TO 60 MIN

Exercises

Problem 7 is the cold solve. No help, no scaffolding, tutor silent, and it is worth protecting the full eight minutes for it. If the hour has run long, cut problem 8 and keep 7. If the hour has run very long, cut the second half of problem 7, parts d and e, and keep the rest.

Hand them the pen and then actually stop talking. This problem contains every error the session was built against, in one place, and watching which one they make is the measurement.

7.

A 6.0 kg crate is released from rest at the top of a ramp inclined at 30 degrees and slides 4.0 m down the ramp. The coefficient of kinetic friction between crate and ramp is 0.25.

(a) Draw the free-body diagram and state, for each force, whether the work it does is positive, negative or zero, and why.
(b) Find the work done by gravity.
(c) Find the work done by friction.
(d) Find the speed of the crate at the bottom of the 4.0 m.
(e) Would a heavier crate arrive faster? Answer with symbols, not numbers.

Original problem, written for this lesson. Geometry after Giancoli Example 6-2 and ch. 6 problem 8.

SOLUTION

(a) Gravity does positive work, because the angle between it and the displacement is 60 degrees, which is under 90. The normal force does zero work, because it is perpendicular to the ramp and the crate moves along the ramp. Friction does negative work, because kinetic friction acts up the ramp while the crate slides down, so the angle is 180 degrees.

(b) W by gravity = mg d sin 30 = (6.0)(9.8)(4.0)(0.500) = 117.6 J, or 118 J to three figures. Keep the 117.6 for part d.

(c) The normal force is mg cos 30 = 50.92 N, so friction is (0.25)(50.92) = 12.73 N and its work is -(12.73)(4.0) = -50.9 J.

(d) Net work = 117.6 - 50.9 = 66.7 J. Starting from rest, one half (6.0) v squared = 66.7, so v = 4.7 m/s.

(e) In symbols the net work is mgd(sin 30 minus 0.25 cos 30), and setting that equal to one half m v squared cancels the m from both sides. Every crate arrives at the same speed. This works only because both forces doing work are proportional to the mass.

COMMON WRONG PATH

Using mg for the normal force on a tilted surface. This is the single highest frequency error in the whole unit and it is quiet, because it produces an answer of the right size. If it appears, do not correct the number. Draw the ramp steeper, close to vertical, and ask how hard the surface is pushing on the crate then.

IF THEY STALL, ASK

“You have three forces. For each one, what is the angle between it and the direction the crate travelled?”

> TUTOR NOTE
> Silence. Do not narrate, do not point, do not react to a wrong first line. Write down which of the four target errors shows up, because that is the note that matters for the next session. What to look for, in order of likelihood. Part c computed with the full weight, friction as (0.25)(6.0)(9.8) = 14.7 N. That gives v = 4.4 m/s, which looks perfectly reasonable, so it will not announce itself. Part b computed as mgd = 235 J, the full weight through the full ramp distance. Friction entered as positive. The normal force credited with work. When they finish, ask for part e before you confirm anything numeric. The mass cancelling is the result worth having and it is the one that comes back in the next three units.

8.

A car of mass m travelling at v-zero on a level road locks its brakes and skids to a stop. The coefficient of kinetic friction is mu. Show that the skid distance is v-zero squared divided by 2 mu g, and say in one sentence why the mass is not in the answer. Then: if the initial speed is 50 percent higher, by what factor does the skid distance grow?

Giancoli, ch. 6, problems 21 and 22, combined and stripped of numbers.

> STRETCH PROBLEM
> Sit with the student if they get stuck rather than rescuing. The struggle here is the point.

SOLUTION

Friction is the only horizontal force and on a level road the normal force is mg, so the friction force is mu m g. It acts opposite the motion over the whole skid, so the net work is minus mu m g d. The car ends at rest, so

minus mu m g d = 0 minus one half m v-zero squared

and d = v-zero squared / (2 mu g).

The mass appears on both sides. A heavier car carries more kinetic energy to remove and it presses harder on the road, so friction removes energy faster in the same proportion. The two effects cancel exactly.

Since d goes as the square of the speed, a factor of 1.5 on the speed is a factor of 2.25 on the distance.

COMMON WRONG PATH

Answering the last part by recomputing with numbers instead of by proportion. It works and it is slower, and on a multiple choice question the proportion is the only route that fits in the time.

IF THEY STALL, ASK

“What is the only horizontal force during the skid, and how much work does it do over a distance d?”

> TUTOR NOTE
> This is the cut candidate. Cut it if the clock is at 56 minutes, because problem 7 part e already carries the mass-cancelling idea and this problem is on the homework as number 8. If you run it, the sentence in the middle is the graded part. Most students will write that the m cancels, which describes the algebra without explaining anything. Push once for the physical version. It is also the sentence that answers why accident investigators can read a speed off a skid mark without knowing what was driving.

The close, two minutes, and it does not get cut. Ask for three things, out loud, no notes. What decides the sign of the work done by a force. What word in the work-energy theorem is doing the most work. Where the theorem came from. The answers you want are: the angle between the force and the displacement. The word net. And it is F equals ma multiplied through by a distance.

Then assign the problem set. One instruction with it: before computing anything on a problem, write the angle between each force and the displacement. Tell them problems 6 and 7 are the two where a classmate is wrong, and that finding the error is the whole answer on those.

ONLY IF THE HOUR IS AHEAD OF PACE | OUTSIDE THE 60

Overflow

Do not start this with fewer than six minutes left. It is the opening of the next lesson and half of it is worse than none of it.

Go back to the incline result: the work done by gravity was mg times the height dropped, and the angle of the ramp did not appear. Ask what that means for two ramps of different steepness that end at the same height. Same work by gravity, every time, regardless of path. Then ask whether the same is true of friction. It is not, because the longer path has more distance for friction to act over.

That split, forces whose work depends only on the endpoints against forces whose work depends on the route taken, is the entire content of the next lesson and the reason potential energy can be defined at all. Stop there. Do not name the words conservative and non-conservative yet; let the observation sit as a question.

9.

A 15 kg backpack is carried at constant speed up a hill of vertical height 10.0 m. (a) How much work does the hiker do on the pack? (b) How much work does gravity do on it? (c) What is the net work? (d) Does any of this depend on how steep the hill is?

Giancoli, Example 6-2, ch. 6, with the numbers rounded.

> STRETCH PROBLEM
> Sit with the student if they get stuck rather than rescuing. The struggle here is the point.

SOLUTION

(a) At constant speed the hiker's upward force equals the weight, 147 N, and the work is 147 times the height, so 1470 J.
(b) Gravity acts downward while the pack rises, so its work is -1470 J.
(c) Zero. The pack leaves at the same speed it started, which is what constant speed means.
(d) No. Only the height enters. A gentle switchback and a vertical ladder cost the hiker the same 1470 J, though not the same effort in any other sense.

COMMON WRONG PATH

Concluding from a net work of zero that the hiker did no work.

IF THEY STALL, ASK

“How far did the pack move in the direction the hiker was pushing?”

TUTOR NOTE

Part c is the one to make them sit with. The hiker did 1470 J of work and the net work was zero. Both statements are true and neither cancels the other. If a student objects that the hiker clearly did something, agree, and point out that where the energy went is exactly the question the next lesson answers.