Work and the work-energy theorem - Answer Key

AP Physics 1 - Unit 3 - Tutor-facing

COURSE AP Physics 1 **UNIT** Unit 3: Work, Energy, and Power
TOPIC Work and the work-energy theorem **SET** U3L1

DIFFICULTY HARD

1. A 15 kg crate is pushed 6.0 m across a level floor by a constant 40 N force directed 25 degrees below the horizontal. (a) Find the work done on the crate by the 40 N force. (b) Find the work done by the normal force and by gravity. (c) Find the magnitude of the normal force.

Fluency. Original problem. The angle is the whole question in part (a). Part (c) is there to see whether a large normal force tempts you into a large amount of work.

SOLUTION (a) The displacement is horizontal and the force sits 25 degrees off it, so $W = Fd\cos\theta = (40)(6.0)\cos 25^\circ = (240)(0.906) = 218$ J, which is 2.2×10^2 J to two figures. (b) Both forces are perpendicular to the displacement, so $\theta = 90^\circ$ and $\cos 90^\circ = 0$. Each does exactly zero work. Not small work. Zero. (c) The crate does not accelerate vertically and the push has a downward component, so $F_N = mg + F\sin 25^\circ = (15)(9.8) + (40)(0.423) = 147 + 16.9 = 164$ N.

WATCH Part (c) is a trap with a purpose. A student who has just computed a 164 N force feels obliged to let it do something. Ask what direction that force points and what direction the crate moved. Also watch for $F_N = mg = 147$ N. The normal force is not a synonym for weight, it is whatever the surface has to push to keep the crate on it.

2. Movers push a 160 kg crate 10.3 m across a rough level floor at constant speed. The effective coefficient of kinetic friction is 0.50. Find (a) the work done by the movers, (b) the work done by friction, and (c) the net work done on the crate. Then state, without computing anything further, how the crate's speed at the end compares with its speed at the start.

Giancoli, ch. 6, problem 4, extended with the net-work part.

SOLUTION The crate does not accelerate vertically, so $F_N = mg$ and $f_k = \mu_k mg = (0.50)(160)(9.8) = 784$ N. It does not accelerate horizontally either, so the push is also 784 N. (a) $W_{push} = (784)(10.3) = 8.1 \times 10^3$ J. (b) Friction points opposite the motion, so $\theta = 180^\circ$ and $W_{fric} = -(784)(10.3) = -8.1 \times 10^3$ J. (c) $W_{net} = 0$. Zero net work means zero change in kinetic energy, so the crate leaves at the same speed it started. That is the constant-speed statement in the problem, arrived at from the other end.

WATCH The check runs both ways and both directions are worth saying out loud. Constant speed forces the net work to be zero, and a net work of zero forces constant speed. A student who writes 8.1×10^3 J for both (a) and (b) with the same sign gets a net work of 16 kJ and a crate that is somehow speeding up while moving at constant speed.

3. How much work must be done to stop a 1250 kg car traveling at 105 km/h?

Giancoli, ch. 6, problem 18.

SOLUTION Convert first: 105 km/h is $105/3.6 = 29.2$ m/s. By the work-energy theorem, $W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = 0 - \frac{1}{2}(1250)(29.2)^2 = -5.32 \times 10^5$ J. The sign is not decoration. Whatever supplies that work, brakes or a wall, pushes opposite to the motion.

WATCH Two failure modes. Leaving the speed in km/h, which produces a number roughly 13 times too big attached to a unit that means nothing. And reporting $+5.32 \times 10^5$ J because stopping feels like it costs effort. Ask which direction the force points relative to the motion, then let them fix the sign.

4. A 140 g baseball traveling at 32 m/s is caught. The fielder's glove moves backward 25 cm while the ball comes to rest in it. (a) Find the work done on the ball. (b) Find the average force the ball exerts on the glove. (c) Now do the problem a second time using only $F = ma$ and the kinematic equations, and say what the work-energy theorem saved you.

Giancoli, ch. 6, problem 20, with an added part (c).

SOLUTION (a) $W = 0 - \frac{1}{2}(0.140)(32)^2 = -71.7$ J on the ball. Negative, because the glove pushes backward on a ball that is moving forward. (b) By Newton's third law the ball pushes forward on the glove while the glove moves forward, so the work done on the glove is $+71.7$ J and $F = W/d = 71.7/0.25 = 2.9 \times 10^2$ N. (c) It can be done that way. The kinematic equation with no time in it is $v^2 = v_0^2 + 2ad$, so $a = \frac{0-(32)^2}{2(0.25)} = -2048$ m/s^2, and $F = ma = (0.140)(2048) = 2.9 \times 10^2$ N. The same answer. What the theorem saved is the acceleration. That -2048 m/s^2 is an intermediate nobody asked for and nothing later depends on, and the theorem is those same two steps with the acceleration already cancelled out of them.

WATCH Part (c) answers the question students never ask, which is why this theorem is on the formula sheet at all when Newton already covers the ground. The honest answer is that it does not cover new ground. It is $F = ma$ multiplied through by a distance, and the payoff is one line instead of two and no stray acceleration to carry. If a student gets -2048 and worries the number is absurd, it is not. A ball stopping in a quarter of a metre from 32 m/s takes about 200 g. Compare it with the case that genuinely closes the Newton route: Giancoli Example 6-5 gives two speeds and no distance and no time, and there $F = ma$ has nothing to stand on.

5. What is the minimum work needed to push a 950 kg car 810 m up a 9.0 degree incline (a) ignoring friction, and (b) if the effective coefficient of friction retarding the car is 0.25?

`Giancoli, ch. 6, problem 10.`

SOLUTION Minimum work means constant speed, so the push balances everything opposing it along the incline. (a) $F_P = mg\sin\theta$, so $W = mgd\sin\theta = (950)(9.8)(810)\sin 9.0^\circ = (7.54\times10^6)(0.156) = 1.2\times10^6$ J. (b) Now $F_P = mg\sin\theta + \mu_k mg\cos\theta$, so $W = mgd(\sin\theta + \mu_k\cos\theta) = (7.54\times10^6)(0.156 + (0.25)(0.988)) = (7.54\times10^6)(0.403) = 3.0\times10^6$ J.

WATCH The standard wrong answer to (a) is $mgd = 7.5\times10^6$ J, the full weight times the full distance along the ramp. It is more than six times too large and it treats a 9 degree ramp as a vertical cliff. If it appears, ask what mgd would mean physically, then ask how high the car actually rose.

6. A 2.0 kg block slides 3.0 m down a ramp inclined at 30 degrees. A classmate reports that the work done by gravity is $W = mgd = (2.0)(9.8)(3.0) = 58.8$ J, and draws an arrow labelled W pointing down the ramp on his free-body diagram. He also says the normal force must do some work, since it acts the whole time the block is moving. Which of his three claims are wrong, and what are the correct values?

`Error catching. Original problem. Three separate errors, one per claim.`

SOLUTION All three are wrong. The 58.8 J: gravity points straight down and the block moves along the ramp, so the angle between them is 60°, not zero. $W_{grav} = mgd\cos 60^\circ = (2.0)(9.8)(3.0)(0.5) = 29.4$ J. Equivalently, only the component $mg\sin 30^\circ$ lies along the motion. Same number either way, since $\cos 60^\circ = \sin 30^\circ$. The arrow: work is a scalar. It has a sign but no direction, so it cannot be drawn as an arrow and it does not belong on a free-body diagram. Only forces go on a free-body diagram. The normal force: it is perpendicular to the ramp surface and the block moves along that surface, so $\theta = 90^\circ$ and $W_N = 0$. Acting for a long time is not the same as doing work.

WATCH The arrow error is the same error as labelling an arrow centripetal force. In both cases a quantity that is not a force gets drawn as one. If a student draws it, do not erase it. Ask what the units of the arrow's length would be.

7. A 2.5 kg block starts from rest on a level floor and is pushed 4.0 m by a constant horizontal 12 N force. Friction on the block is a constant 4.0 N. A classmate computes $W_{net} = (12)(4.0) + (4.0)(4.0) = 64$ J and reports a final speed of 7.2 m/s. Is he right? If not, give the correct speed, and describe one check he could have run on his own answer that would have caught the mistake without redoing the problem.

Error catching. Original problem. The friction sign, and a way to notice it.

SOLUTION He is wrong. Friction opposes the motion, so it does negative work: $W_{fric} = -(4.0)(4.0) = -16$ J. Then $W_{net} = 48 - 16 = 32$ J, and $\frac{1}{2}(2.5)v^2 = 32$ gives $v = \sqrt{25.6} = 5.1$ m/s. The check: remove friction entirely and the net work is 48 J, giving $v = \sqrt{2(48)/2.5} = 6.2$ m/s. His answer of 7.2 m/s is faster than the block would go on a frictionless floor. A rough floor cannot make a block faster than a smooth one, so the answer was wrong before any of the algebra was rechecked.

WATCH The reason this error survives is that nothing about 64 J looks broken. Every number is positive, the units are joules, the arithmetic is right. Building the frictionless-ceiling check into the habit is worth more here than the correction itself. Ask for the check before asking for the number.

8. A car of mass m traveling at speed v_0 on a level road locks its brakes and skids to a stop. The coefficient of kinetic friction between tires and road is μ_k. Use no numbers. (a) Show that the skid distance is $d = v_0^2/(2\mu_k g)$. (b) State in one sentence why the mass does not appear in the answer. (c) If the initial speed is increased by 50 percent and nothing else changes, by what factor does the skid distance change?

Symbolic. Giancoli ch. 6, problems 21 and 22, combined and stripped of numbers.

SOLUTION (a) The only horizontal force is friction, and on a level road $F_N = mg$, so $f_k = \mu_k mg$. It points opposite the motion over the whole skid, so $W_{net} = -\mu_k mgd$. The car ends at rest, so $-\mu_k mgd = 0 - \frac{1}{2}mv_0^2$, and therefore $d = \frac{v_0^2}{2\mu_k g}$. (b) The mass sits on both sides. A heavier car carries more kinetic energy to remove, and it presses harder on the road, so friction removes energy faster in exactly the same proportion. The two effects cancel. (c) d goes as v_0^2, so multiplying the speed by 1.5 multiplies the distance by $1.5^2 = 2.25$.

WATCH Part (b) is the part to grade hardest. Most students will write that the mass cancels, which restates the algebra rather than explaining it. Push for the physical sentence: more energy to remove, and a proportionally larger friction force removing it. Part (c) is where a student who thinks in proportions beats a student who recomputes.

9. A block of mass m is pulled a distance d across a level floor by a constant force F directed at an angle θ above the horizontal. The coefficient of kinetic friction is μ_k. Using symbols only, write expressions for the work done by (a) F, (b) the normal force, (c) gravity, (d) friction, and (e) the net force. (f) Holding the magnitude of F fixed and increasing θ from zero, does the work done by F increase, decrease, or stay the same? (g) Does the net work behave the same way? Explain.

Symbolic. Giancoli, ch. 6, Exercise A, extended to the full force inventory.

SOLUTION (a) $W_F = Fd\cos\theta$. (b) $W_N = 0$, perpendicular to the motion. (c) $W_{grav} = 0$, also perpendicular. (d) The block does not accelerate vertically, so $F_N = mg - F\sin\theta$, and $W_{fric} = -\mu_k(mg - F\sin\theta)d$. (e) $W_{net} = Fd\cos\theta - \mu_k(mg - F\sin\theta)d$. (f) It decreases. $\cos\theta$ falls from 1 toward 0 as θ goes from 0 to 90°, so less and less of F lies along the motion. (g) No. Tilting the force upward also lifts some of the block's weight off the floor, which shrinks F_N and makes the friction term less negative. One term falls while the other rises, so the net work can increase at first and fall later. Which effect wins depends on μ_k.

WATCH Part (d) is where the problem earns its place. Writing $F_N = mg$ out of habit is the single most common line here, and it is what makes part (g) invisible. If a student answers (g) with a flat decrease, send them back to their own expression for F_N rather than telling them the answer.

10. A softball of mass 0.25 kg is pitched at 95 km/h. By the time it reaches the plate 15 m away it has slowed by 10 percent. Neglecting gravity, estimate the average force of air resistance on the ball during the pitch.

Giancoli, ch. 6, problem 23.

SOLUTION $v_0 = 95/3.6 = 26.4$ m/s and $v = 0.90(26.4) = 23.8$ m/s. $W_{net} = \frac{1}{2}(0.25)[(23.8)^2 - (26.4)^2] = (0.125)(564 - 696) = -16.5$ J. Air resistance is the only horizontal force and it acts opposite the motion over the full 15 m, so $-F(15) = -16.5$ J and $F = 1.1$ N.

WATCH Watch for the student who takes 10 percent off the kinetic energy instead of off the speed. Kinetic energy goes as v^2, so a 10 percent speed drop is a 19 percent energy drop. Watch also for the ball's weight, about 2.5 N, being added in. The problem says to neglect gravity, and gravity is perpendicular to the flight anyway.

11. A 285 kg load is lifted 22.0 m vertically by a single cable, starting from rest, with a constant upward acceleration of $0.160g$. Find (a) the tension in the cable, (b) the net work done on the load, (c) the work done by the cable, (d) the work done by gravity, and (e) the final speed of the load. (f) A classmate says the work done by the cable should equal the change in kinetic energy. Explain in one sentence why it does not.

`Giancoli, ch. 6, problem 25, with an added part (f).`

SOLUTION (a) Taking up as positive, $F_T - mg = ma = 0.160mg$, so $F_T = 1.160mg = (1.160)(285)(9.80) = 3.24 \times 10^3$ N. (b) $F_{net} = 0.160mg$ and it is parallel to the motion, so $W_{net} = (0.160)(285)(9.80)(22.0) = 9.83 \times 10^3$ J. (c) $W_{cable} = F_T d = (3.24 \times 10^3)(22.0) = 7.13 \times 10^4$ J. (d) Gravity is opposite the motion, so $W_{grav} = -mgd = -(285)(9.80)(22.0) = -6.14 \times 10^4$ J. Check: $7.13 \times 10^4 - 6.14 \times 10^4 = 9.9 \times 10^3$ J, which agrees with (b) once the rounding in the intermediate values is allowed for. (e) $\frac{1}{2}(285)v^2 = 9.83 \times 10^3$, so $v = \sqrt{69.0} = 8.31$ m/s. (f) The theorem is about the work done by all the forces. The cable is one of two. Gravity removes most of what the cable supplies, and only the difference shows up as kinetic energy.

WATCH Part (f) is the standing misuse of this theorem and it will recur every time energy appears this year. Watch also for (b) being computed as W_{cable}, which is the same error in a different costume. The internal check between (b), (c) and (d) is worth requiring as part of the answer rather than as an afterthought.

12. Free response. A 6.0 kg crate is released from rest at the top of a ramp inclined at 30 degrees above the horizontal and slides 4.0 m down the ramp. The coefficient of kinetic friction between crate and ramp is 0.25. (a) Draw a free-body diagram for the crate. For each force on it, state whether the work it does is positive, negative or zero, and justify each answer by reference to the angle between that force and the displacement. (b) Calculate the work done by gravity. (c) Calculate the work done by the normal force. (d) Calculate the work done by friction. (e) Calculate the speed of the crate at the bottom of the 4.0 m. (f) Without using numbers, show that the speed at the bottom does not depend on the mass of the crate, and state the one feature of this situation that makes that true.

AP-style free response. Original problem, built on the geometry of Giancoli example 6-2 and ch. 6 problem 8.

SOLUTION (a) Three forces. Gravity does positive work: it points straight down, the crate moves down the ramp, and the angle between them is 60°, which is less than 90°. The normal force does zero work: it is perpendicular to the ramp surface and the crate moves along that surface, so $\theta = 90^\circ$. Friction does negative work: the crate slides down and kinetic friction acts up the ramp, so $\theta = 180^\circ$. (b) $W_{grav} = mgd\sin 30^\circ = (6.0)(9.8)(4.0)(0.500) = 117.6$ J, which is 118 J to three figures. Carry the 117.6 into part (e). (c) $W_N = 0$. (d) $F_N = mg\cos 30^\circ = (6.0)(9.8)(0.866) = 50.92$ N, so $f_k = (0.25)(50.92) = 12.73$ N and $W_{fric} = -(12.73)(4.0) = -50.9$ J. (e) $W_{net} = 117.6 - 50.9 = 66.7$ J. Starting from rest, $\frac{1}{2}(6.0)v^2 = 66.7$, so $v = \sqrt{22.2} = 4.7$ m/s. (f) In symbols, $W_{net} = mgd\sin\theta - \mu_k mgd\cos\theta = mgd(\sin\theta - \mu_k\cos\theta)$. Setting that equal to $\frac{1}{2}mv^2$ and cancelling m from both sides gives $v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$, with no mass in it. This works only because every force that does work here is proportional to the mass, gravity directly and friction through the normal force. Add one force that is not, air resistance for instance, and the mass stops cancelling.

WATCH Score it the way the AP would. Part (a) carries as many points as any calculation, and a student who writes the three signs without naming an angle has answered a different question. In (d) the standard error is $f_k = \mu_k mg = 14.7$ N, which uses the full weight on a tilted surface and produces $v = 4.4$ m/s. Close enough to look right, which is what makes it dangerous. In (f), require the final sentence. Naming the condition under which a result holds is the difference between a 4 and a 5.